Conservative vector fields

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* Recall: a vector field F is said to be conservative if

$$F = \nabla f$$

for some function f. The function f is called the potential of the v.f.

Ex $F = \langle 3x^2 + y^2, 2xy + 2y \rangle$

Is it a conservative vector field ?

$F = \nabla f = \langle f_x, f_y \rangle$.

$f_x = 3x^2 + y^2 \rightsquigarrow f(x,y) = x^3 + xy^2 + C(y)$

$f_y = 2xy + C'(y)$

$\rightsquigarrow C'(y) = 2y$

$\rightsquigarrow C(y) = y^2 + D$

$f(x,y) = x^3 + xy^2 + y^2 + D$

Ex $F(x,y) = \langle y^2, x^2 \rangle$

Is it a conservative vector field ?

$y^2 = f_x \rightsquigarrow f = xy^2 + C(y)$

$f_y = 2xy + C'(y) = x^2$

$\rightsquigarrow$ there is no such f !